

Scattering Ripples from Branes

STEVEN B. GIDDINGS[†]

*Department of Physics
University of California
Santa Barbara, CA 93106-9530*

Abstract

A novel probe of D-brane dynamics is via scattering of a high energy ripple traveling along an attached string. The inelastic processes in which the D-brane is excited through emission of an additional attached string is considered. Corresponding amplitudes can be found by factorizing a one-loop amplitude derived in this paper. This one-loop amplitude is shown to have the correct structure, but extraction of explicit expressions for the scattering amplitudes is difficult. It is conjectured that the exponential growth of available string states with energy leads to an inclusive scattering rate that becomes large at the string scale, due to excitation of the “string halo,” and meaning that such probes do not easily see structure at shorter scales.

[†] Email address: giddings@denali.physics.ucsb.edu

1. Introduction

Following the string revolution of ca. 1984, theoretical physics was in the unique situation of having a candidate for the “theory of everything,” namely the heterotic string. The ongoing string revolution of the 1990’s has changed that picture considerably, particularly as a result of the discovery of the importance of D-brane states¹ and dualities between different vacua [2]. One question that no longer apparently has a clear answer is that of describing the fundamental degrees of freedom of the theory we are studying.

In particular, the relative roles of D-brane states and string states are uncertain. Although D-branes seem to be solitons of string theory, it is not obvious that they are solitons in precisely the same sense as other more familiar solitons in field theory. A useful analogy is the role of monopoles in QED versus that in a grand unified theory. There is a consistent set of rules, first introduced by Dirac, that provides a treatment of many of the physical phenomena in the presence of a monopole within QED. However, there are certain questions that don’t have answers in QED, such as the scattering behavior of Callan-Rubakov modes or the amplitude for high-energy e^+e^- scattering to produce a monopole anti-monopole pair. In contrast, in grand unified theories, monopoles are true solitons in the sense that they can be constructed smoothly from quanta of the underlying gauge and Higgs fields.² This fact allows explicit treatment of both the Callan-Rubakov modes and of monopole production, and all questions are in principle answerable in terms of the Yang-Mills/Higgs dynamics of the fundamental fields.

It is far from clear that D-branes are solitons of string theory in the same sense. For one thing, D-branes carry fundamental charge, namely Ramond-Ramond charge, not carried by the string states. It is not immediately apparent that a D-brane can be smoothly assembled from underlying string degrees of freedom. In fact, at least two alternatives have been proposed in the literature. The first[3] is that there is a sort of democracy, namely strings are usefully treated as the fundamental degrees of freedom in some regions of the moduli space of theories, and D-branes are most usefully taken as fundamental in other regions. The two descriptions would then be patched together at intermediate moduli. A second[4] is that the D0-branes are fundamental constituents of the theory from which other states can be assembled.

¹ See [1] for recent reviews.

² More precisely, widely separated monopole-antimonopole pairs can be so created.

To give this question a sharp edge it is useful to turn to scattering problems. A crucial sharp question that doesn't appear to have a good answer in the present formalism is that of computing the amplitude for a high-energy annihilation of a pair of strings to create a pair consisting of a D0-brane and its antiparticle. This type of problem may well push the first alternative above past its limits, and drives towards the heart of the question of what the fundamental degrees of freedom are.³

It would also be useful to investigate other types of scattering phenomena to search for other problems analogous to the pair production problem, or to the Callan-Rubakov problem in QED, where the scattering description for certain modes manifestly breaks down. Furthermore, given evidence for structure at distances below the string scale [6], a related problem is that of looking for scattering phenomena that further exhibit and illuminate such short-distance structure.

Various types of scattering phenomena involving branes have been considered. Refs. [7-8] treated scattering of strings from branes. Except in the case of D-instantons, this type of scattering is apparently dominated by dynamics at the string scale. Brane-brane scattering has been considered in [9-11], and although it has revealed evidence of distances at shorter scales, is somewhat difficult to treat in generality.

The purpose of this paper is to begin an investigation of another type of scattering involving branes.⁴ Consider a long string stretched between two widely separated branes. One can attempt to probe the structure of one of the branes by sending a ripple down the string to collide with the brane. Furthermore, in [1] it was argued that such scattering might be capable of revealing structure at sub-stringy scales. To see this, consider working at weak string coupling g . To leading order, the ripple is simply reflected due to the Dirichlet boundary conditions at the end of the string. This corresponds to a constant (frequency independent) phase shift, and is indicative of point-like structure. Before jumping to conclusions, however, one should consider higher order processes such as one where a ripple collides with a brane and produces an excitation of the brane. These processes first enter at order g in perturbation theory. Suppose for example that the inclusive amplitude for such processes grew as

$$\mathcal{A} \sim g\omega^p ,$$

³ Banks and Susskind [5] raised the closely related puzzle of what happens when a D0 pair annihilate, and give arguments for the current intractability of this problem.

⁴ This type of scattering was first described in [1].

for some p , at high frequency. This amplitude only becomes substantial, indicating possible breakdown of pointlike structure, at frequencies of order $g^{-1/p}$. This would indicate that such scattering is governed by dynamics at shorter scales than the string scale.

This paper will begin an investigation of higher order processes with the order g amplitude. This will be treated by applying the optical theorem to, or equivalently factorizing, the appropriate one-loop amplitude. Section two will discuss the kinematical setup and related issues. Section three derives an expression for this amplitude, and the following section attempts an analysis of this amplitude in the open-string factorization limit. Unfortunately the resulting expressions have not been sufficiently tractable to extract a definitive answer about the high-energy scattering behavior. However, as argued in section five, one plausibly discovers high-energy amplitudes that grow exponentially in the energy. This behavior could result from the exponential growth of the number of available final states. If this is the correct behavior, it means that the scattering becomes large at least before a scale only logarithmically down from the string scale, and suggests that structure beyond the string scale is difficult to probe by scattering ripples from branes.

The appendix collects some formulas useful for computing one-loop amplitudes.

2. Groundwork

Our goal is to study scattering of a high-energy pulse, traveling along a string, off of an attached D-brane. To simplify the calculation, I'll begin by considering such a pulse traveling on a string stretched between two parallel branes with separation L , and then take the limit $L \rightarrow \infty$.

Creating a high frequency state on the string means creating a state with a level number scaling like L as $L \rightarrow \infty$. To be precise, let $|0, L\rangle$ be the vacuum in the sector of strings connecting the two branes. Acting with α_{-n}^μ produces an oscillation of frequency

$$\omega = \frac{\pi n}{L}. \quad (2.1)$$

If we consider the mass-shell condition (with $\alpha' = 1/2$)

$$M^2 = \frac{L^2}{\pi^2} + 2(n-1) \quad (2.2)$$

in the limit $L \rightarrow \infty$, we indeed see that the rest frame energy is shifted by ω :

$$E_{cm} = M = \frac{L}{\pi} + \omega + \mathcal{O}\left(\frac{1}{L}\right). \quad (2.3)$$

However, acting with α_{-n}^μ does not produce a physical state satisfying the Virasoro conditions

$$(L_0 - 1) |\text{phys}\rangle = L_n |\text{phys}\rangle = 0 \quad (n > 0) . \quad (2.4)$$

This raises the possibility of spurious behavior from the unphysical part of the operator. This potential pitfall can be avoided by working only with physical states, for example by working in the light-cone gauge. With D-branes, however, there is a new subtlety. Light cone gauge corresponds to picking out a null combination $X^+ = X^0 + X^1$ and identifying it with world-sheet time,

$$X^+ \propto \tau. \quad (2.5)$$

In particular, note that this implies X^+ satisfies Neumann boundary conditions

$$n^a \nabla_a X^+ = 0 \quad (2.6)$$

on the world-sheet boundary. Therefore X^0 and X^1 can't be transverse to the brane, and our treatment applies only to p-branes with $p \geq 1$.⁵ Coordinates will therefore be taken to be X^i , transverse to the brane, and $X^I = \{X^\pm, X^a\}$, tangent to the brane.

Rather than working directly in the light-cone gauge, I'll instead work in the covariant framework using the DDF operators[12] to translate the physical states from light cone gauge. To construct these, let p_0 be a momentum satisfying the unexcited mass-shell condition

$$p_0^2 = - \left(\frac{L}{\pi} \right)^2 + 2 \quad (2.7)$$

and choose a null vector k (defining the directions X^\pm) from the subspace tangent to the brane, such that

$$k \cdot p_0 = 1. \quad (2.8)$$

The DDF operators are then⁶

$$\begin{aligned} A_{-n}^a &= \frac{1}{2\pi} \int_0^{2\pi} d\tau \, \dot{X}^a e^{-ink \cdot X} \\ A_{-n}^i &= \frac{1}{2\pi} \int_0^{2\pi} d\tau \, X^{i'} e^{-ink \cdot X} \end{aligned} \quad (2.9)$$

⁵ Note that this adds a subtlety to the usual approach of cataloging physical states, based on the light-cone frame, for the case of strings attached to D-particles or D-instantons.

⁶ See [13] for elaboration on the properties of DDF operators.

for oscillators parallel or transverse to the brane, respectively. Here the integrals are carried out along the world-sheet boundary, which we will assume is attached to one of the branes.

To summarize, the picture is as follows. We have a string attached to two parallel branes, one through the origin and one at a distance L . We act with a DDF operator A_{-n}^a or A_{-n}^i at the boundary attached to the brane at L . This creates a plane wave-like oscillation of the string of energy $\omega = \pi n/L$ satisfying the Virasoro conditions and the mass-shell condition (2.2). We then wish to consider the limit $L \rightarrow \infty$, holding ω fixed and large, and determine what final states are produced in scattering the oscillation, or ripple, off the brane.

This is a difficult general problem so I'll specialize further. In particular, this paper will focus on the one-string emission amplitude, as this can be found by factorizing the one loop amplitude. For further simplicity I'll also consider only polarizations parallel to the brane, although many of the formulas translate directly to polarizations perpendicular to both the brane and the direction separating the branes.

3. The One-Loop Amplitude.

As stated above, inclusive scattering with one final string can be studied by factorizing the one-loop amplitude. To be precise, we consider the four-point function on the annulus, as shown in fig. 1. Two of the external states, created by operators \mathcal{O}_1 and \mathcal{O}_4 , correspond to the ground state of the string stretched between the branes,

$$\begin{aligned}\mathcal{O}_1 &= e^{ip_0 \cdot X} \mathcal{O}_D(\tau_1) \\ \mathcal{O}_4 &= e^{-ip_0 \cdot X} \mathcal{O}_D(\tau_4).\end{aligned}\tag{3.1}$$

Here \mathcal{O}_D are operators that create string states with the appropriate Dirichlet boundary conditions. The other two operators are DDF operators (de)exciting the strings,

$$\begin{aligned}\mathcal{O}_2 &= A_{-n}^a \\ \mathcal{O}_3 &= A_n^a.\end{aligned}\tag{3.2}$$

These operators may be taken arbitrarily close to \mathcal{O}_1 and \mathcal{O}_4 , respectively. The processes

$$\text{ripple} + \text{brane} \rightarrow \text{ripple} + \text{excited brane}$$

can be investigated by factorizing this amplitude in the open string channel, $\lambda \rightarrow 0$. At intermediate times, the excitation of the brane arises from an open string with both ends attached to the brane.

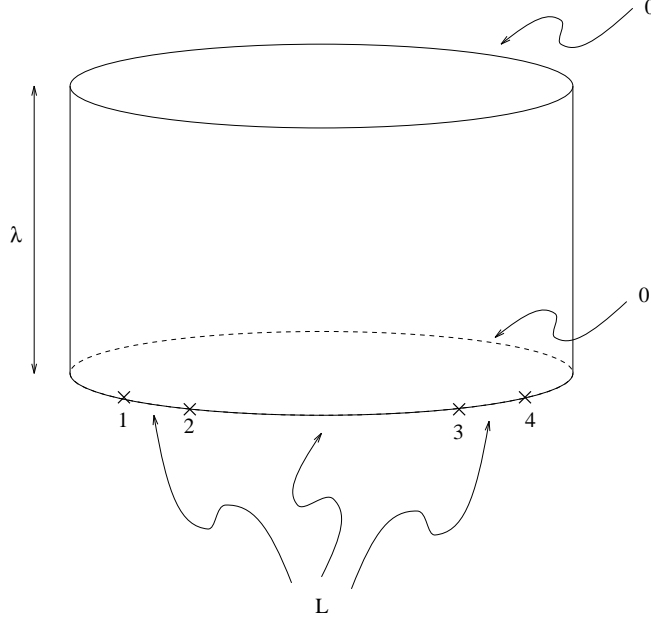


Fig. 1: In this one-loop diagram, a string stretching between the branes at 0 and L is created at point 1. This is excited by a DDF operator at 2, acting on the boundary of the string at L . These states are likewise annihilated at 3 and 4. The upper boundary of the annulus is fixed on the brane at 0. The annulus is taken to have circumference 1 and height λ . Factorizing in the $\lambda \rightarrow 0$ limit, we find the amplitude for the oscillation of the long string to create an excitation of the brane at 0 involving a string with both its ends attached to the brane.

The one-loop amplitude is given by the standard Polyakov formula,

$$\mathcal{A} = \int d\mu(m_i) \int \mathcal{D}X^I \mathcal{D}X^i e^{-S} \mathcal{O}_1 \mathcal{O}_2 \mathcal{O}_3 \mathcal{O}_4 \quad (3.3)$$

where $d\mu(m_i)$ is the measure for moduli and

$$S = \frac{1}{2\pi} \int d^2\sigma \sqrt{g} (\nabla X)^2. \quad (3.4)$$

The integral over X^\pm is straightforward to evaluate by standard methods, and gives

$$\left(\frac{\det'_N \Delta}{\int d^2\sigma \sqrt{g}} \right)^{-1} \exp \left\{ \frac{1}{4} \sum_{i,j} P_i \cdot P_j G_N(\tau_i, \tau_j) \right\} \quad (3.5)$$

where the determinant is taken with Neumann boundary conditions, prime denotes omission of the zero mode, P_i represent the four external state momenta, and

$$G_N(\tau_i, \tau_j | \lambda) = 2 \ell n \left[\frac{\theta_1(\tau_i - \tau_j | 2i\lambda)}{\theta_1(\tau_i - \tau' | 2i\lambda)} \right] \quad (3.6)$$

is the Neumann Green function (see appendix). Eq. (3.5) becomes

$$\frac{\int d^2\sigma \sqrt{g}}{\det'_N \Delta} \left[\frac{\theta_1(\tau_1 - \tau_3|2i\lambda)\theta_1(\tau_2 - \tau_4|2i\lambda)}{\theta_1(\tau_1 - \tau_2|2i\lambda)\theta_1(\tau_3 - \tau_4|2i\lambda)} \right]^n \theta_1(\tau_1 - \tau_4|2i\lambda)^{-p_0^2} \theta_1(\epsilon|2i\lambda)^{p_0^2} \quad (3.7)$$

where ϵ is a worldsheet UV regulator. Likewise the integral over X^a gives

$$\left(\frac{\det'_N \Delta}{\int d^2\sigma \sqrt{g}} \right)^{-(p-1)/2} [\ell n \theta_1]''(\tau_2 - \tau_3). \quad (3.8)$$

The integral over X^i involves the operators \mathcal{O}_D . We need not treat these explicitly, but rather note that they create states satisfying the Dirichlet boundary conditions

$$\begin{aligned} X &= 0 & \tau > \tau_4, \tau < \tau_1 \\ X &= L & \tau_4 > \tau > \tau_1 \end{aligned} \quad (3.9)$$

in the coordinate separating the branes.

A general formula for the functional integral with Dirichlet boundary conditions

$$X|_{\partial} = x(\tau) \quad (3.10)$$

is

$$\begin{aligned} \int_{X|_{\partial}=x(\tau)} \mathcal{D}X e^{-S} = \\ \int_{X|_{\partial}=0} \mathcal{D}X e^{-S} \exp \left\{ -\frac{1}{4\pi^2} \oint d\tau \oint d\tau' x(\tau)x(\tau') \partial_n \partial_{n'} G_D(\tau, \tau') \right\} \end{aligned} \quad (3.11)$$

where the normal derivatives ∂_n act on the Dirichlet Green function (see appendix)

$$G_D(z, w|\lambda) = \frac{1}{2} \ell n \left[\frac{\theta_1(z - w|2i\lambda)\theta_1(\bar{z} - \bar{w}|2i\lambda)}{\theta_1(z - \bar{w}|2i\lambda)\theta_1(\bar{z} - w|2i\lambda)} \right] + \frac{2\pi}{\lambda} \text{Im} w \text{Im} z. \quad (3.12)$$

From this we derive the result for the integral over X^a ,

$$\det_D \Delta^{-(d-p-1)/2} \theta_1(\tau_4 - \tau_1|2i\lambda)^{-L^2/\pi^2} e^{-L^2(\tau_1 - \tau_4)^2/2\pi\lambda} \theta_1(\epsilon|2i\lambda)^{L^2/\pi^2}, \quad (3.13)$$

where the determinant is taken with Dirichlet boundary conditions and $d = 26$ for the bosonic string.

The determinants are readily evaluated (see appendix), and give

$$\begin{aligned}\frac{det'_N \Delta}{\int d^2 \sigma \sqrt{g}} &= 2q^{1/12} f(q^2) \\ det_D \Delta &= 2\lambda q^{1/12} f(q^2)\end{aligned}\tag{3.14}$$

with $f(q^2) = \prod_{n=1}^{\infty} (1 - q^{2n})$ and $q = e^{-2\pi\lambda}$, and these combine with the measure into the result

$$\begin{aligned}d\mu(m_i) &\left(\frac{det'_N \Delta}{\int d^2 \sigma \sqrt{g}} \right)^{-(p+1)/2} (det'_D \Delta)^{-(d-p-1)/2} \\ &= d\lambda d\tau_4 \lambda^{-(26-p-1)/2} 2^{-12} q^{-2} f^{-24}(q^2).\end{aligned}\tag{3.15}$$

Finally, note that the integrals in the DDF operators (2.9) involve the coordinate τ intrinsic to the infinite strip string world sheet corresponding to string propagation. When this is continued to euclidean space and mapped to a portion of the annulus, using $z = e^{i\tau+i\sigma}$, with the incoming state mapped to a point, then the integral is transformed to one over a coordinate z that circles the incoming state:

$$\int_0^{2\pi} d\tau \longrightarrow \oint_{z=z_0} dz.\tag{3.16}$$

Collecting these formulas, setting $\tau_1 = 0$, renaming $\tau_4 = \tau$, and dropping an overall infinite factor, we arrive at the expression for the desired one loop amplitude:

$$\mathcal{A} = \int_0^\infty d\lambda \int_0^1 d\tau \lambda^{-(26-p-1)/2} 2^{-12} q^{-2} f^{-24}(q^2) \left[\frac{\theta_1(\tau|2i\lambda)}{\theta'_1(0|2i\lambda)} \right]^{-2} e^{-L^2 \tau^2 / 2\pi \lambda} \mathcal{C} \tag{3.17}$$

where

$$\begin{aligned}\mathcal{C} &\equiv \frac{1}{n} \oint_{z_1=0} \frac{dz_1}{2\pi} \oint_{z_2=\tau} \frac{dz_2}{2\pi} \\ &\left[\frac{\theta_1(\tau - z_1|2i\lambda) \theta_1(\tau - z_2|2i\lambda)}{\theta_1(z_1|2i\lambda) \theta_1(z_2|2i\lambda)} \right]^n [\ell n \theta_1]''(\tau - z_1 - z_2|2i\lambda)\end{aligned}\tag{3.18}$$

and the extra factor of $1/n$ arises from normalizing the DDF operators to unity.

Recall that we wish to investigate the expression as $n, L \rightarrow \infty$. One might have hoped this limit yields a simple expression, as this is the apparently simple limit where one of the branes is irrelevant and we simply have a plane wave propagating on a semi-infinite string. Unfortunately, simplification does not obviously occur. From the physical standpoint, other effects besides scattering from the brane complicate matters. In particular, if one looks at the annulus diagram in the closed string channel, it describes exchange of a graviton between the brane and the now infinitely massive string. Instead of analyzing the general expression, the next section will consider it in the open-string factorization limit, $\lambda \rightarrow 0$.

4. Open String Factorization

In order to investigate the expression (3.17) we will consider it in the open string factorization limit, $\lambda \rightarrow 0$. In this limit it is convenient to use the modular transformation properties of the theta functions to rewrite them as functions of

$$\tilde{q} = e^{-\pi/\lambda}. \quad (4.1)$$

It is also convenient to define variables

$$s = e^{-\pi\tau/\lambda} \quad (4.2)$$

and

$$w_i = e^{\pi z_i/2\lambda}. \quad (4.3)$$

The various contributions to (3.17) are readily expanded in terms of these variables (as we'll see momentarily this leads to a direct physical interpretation):

$$\begin{aligned} \frac{\theta(\tau - z_i|2i\lambda)}{\theta(z_i|2i\lambda)} &= e^{-\pi\tau^2/2\lambda} w_i^{2\tau} \sqrt{\frac{1}{s}} \frac{w_i^{-1} - s w_i}{w_i - w_i^{-1}} \\ &\prod_{n=1}^{\infty} \frac{(1 - \tilde{q}^n/s w_i^2)(1 - \tilde{q}^n s w_i^2)}{(1 - \tilde{q}^n/w_i^2)(1 - \tilde{q}^n w_i^2)}, \\ [\ell n \theta_1]''(\tau - z_1 - z_2|2i\lambda) &= -\frac{\pi}{\lambda} - \left(\frac{\pi}{\lambda}\right)^2 \left[\frac{s}{(1/w_1 w_2 - s w_1 w_2)^2} \right. \\ &\quad \left. + \sum_{n=1}^{\infty} \frac{\tilde{q}^n/s w_1 w_2}{(1 - \tilde{q}^n/s w_1 w_2)^2} + \frac{\tilde{q}^n s w_1 w_2}{(1 - \tilde{q}^n s w_1 w_2)^2} \right], \end{aligned} \quad (4.4)$$

and

$$\left[\frac{\theta_1(\tau)}{\theta_1'(0)} \right]^{-2} = \frac{\pi^2}{\lambda^2} e^{\pi\tau^2/\lambda} \frac{s}{(1-s)^2} \prod_{n=1}^{\infty} \frac{(1 - \tilde{q}^n)^4}{(1 - \tilde{q}^n s)^2 (1 - \tilde{q}^n/s)^2}. \quad (4.5)$$

We also use

$$2^{-12} q^{-2} f^{-24}(q^2) = \lambda^{12} \tilde{q}^{-1} f(\tilde{q})^{-24} = \lambda^{12} \sum_{\ell=0}^{\infty} d_{\ell} \tilde{q}^{\ell-1}, \quad (4.6)$$

where d_{ℓ} is the degeneracy of open string states at level ℓ .

The preceding expressions imply the existence of an expansion of \mathcal{C} in s and \tilde{q} , of the form

$$\begin{aligned} \left[\frac{\theta_1(\tau)}{\theta_1'(0)} \right]^{-2} \mathcal{C} &= e^{-\pi\tau^2(n-1)/\lambda} \\ &\sum_{m=0}^{\infty} \sum_{\ell=-m-n}^{\infty} \left[\frac{2}{\pi\lambda} G_{\ell m}^1(n\tau, n) + \frac{1}{\lambda^2} G_{\ell m}^2(n\tau, n) \right] s^{\ell+1} \tilde{q}^m. \end{aligned} \quad (4.7)$$

This expression is readily interpreted after inserting into the amplitude (3.17),

$$\mathcal{A} = \sum_{r=0}^{\infty} d_r \sum_{m=0}^{\infty} \sum_{\ell=-m-n}^{\infty} \int_0^{\infty} d\lambda \int_0^1 d\tau \lambda^{\frac{p+1}{2}-3} \left[\frac{2\lambda}{\pi} G_{\ell m}^1(n\tau, n) + G_{\ell m}^2(n\tau, n) \right] \exp \left\{ -\frac{L^2 \tau^2}{2\pi\lambda} - \frac{\pi(n-1)\tau^2}{\lambda} - \frac{\pi\tau}{\lambda}(\ell+1) - \frac{\pi}{\lambda}(m+r-1) \right\}. \quad (4.8)$$

After defining $T = \frac{\pi}{2\lambda}$, we find

$$\mathcal{A} \propto \sum_{r=0}^{\infty} d_r \sum_{m=0}^{\infty} \sum_{\ell=-m-n}^{\infty} \int_0^{\infty} \frac{dT}{T^{(p+1)/2-1}} \int_0^1 d\tau \left[\frac{1}{T} G_{\ell m}^1(n\tau, n) + G_{\ell m}^2(n\tau, n) \right] \exp \left\{ -T\tau^2 \left[\frac{L^2}{\pi^2} + 2(n-1) \right] - 2(\ell+1)T\tau - 2(m+r-1)T \right\}. \quad (4.9)$$

Ignoring the $G_{\ell m}^i$'s, this is the Schwinger proper time formula for the one loop amplitude of a particle with $M^2 = \frac{L^2}{\pi^2} + 2(n-1)$ splitting into two particles with masses given by

$$\begin{aligned} \mu_1^2 &= 2(m+r-1) \\ \mu_2^2 &= M^2 + 2(m+r+\ell). \end{aligned} \quad (4.10)$$

The $G_{\ell m}^i$'s are effective coupling functions for these decays.

The details of the kinematics are easily seen in the $L \rightarrow \infty$ limit. Recall from (2.3),

$$M = \frac{L}{\pi} + \omega. \quad (4.11)$$

Similarly, (4.10) gives

$$\mu_2 = M + \frac{\pi}{L}(m+r+\ell), \quad (4.12)$$

corresponding to a final state frequency for the long string

$$\omega - \Delta\omega \quad (4.13)$$

with

$$-\Delta\omega = \frac{\pi}{L}(m+r+\ell). \quad (4.14)$$

The threshold condition for cuts corresponding to decay is $M > \mu_1 + \mu_2$, or

$$\Delta\omega \geq \sqrt{2(m+r-1)}. \quad (4.15)$$

The interpretation of such cuts is clear: the infinite string loses energy $\Delta\omega$ by emitting an open string at mass level $m+r$, with both of its ends attached to the brane.

There are three classes of such states. The first, the tachyonic state, is an unphysical artifact and will be ignored. The massless states correspond to elastic motion of the brane—either center of mass recoil or tension-driven vibrations of the brane, along with world volume gauge field excitations. For $\omega < \sqrt{2}$, (4.15) shows that these are the only modes excited. For $\omega > \sqrt{2}$, one also excites massive string modes. These “string halo” excitations may or may not be properly thought of as internal structural degrees of freedom of the brane.

5. High energy structure

Although such “halo” modes have a threshold corresponding to the string scale, one would like to know at what energy scale the scattering amplitudes become large. The relevance of this was alluded to in the introduction: at $\mathcal{O}(g)$ in the coupling constant, the scattering has a constant phase shift, and thus appears sensitive to the point-like structure of the D-brane. Including scattering effects at $\mathcal{O}(g)$ and higher, we wish to determine the scale at which the scattering becomes non-trivial. As suggested in the introduction, power law behavior

$$\text{Im } \mathcal{A} \propto g^2 \omega^{2p} \tag{5.1}$$

in the imaginary part of the one-loop amplitude would suggest that the relevant scale is $E \sim g^{-1/p}$, exhibiting structure on sub-string distances.

A definitive answer to this question apparently requires determination of the effective couplings $G_{\ell m}^i$. So far this has not been possible. However, there is one clue to the large- ω behavior of the amplitude. At large ω , the number of possible states for the radiated string grows as $e^{\sqrt{8}\pi\omega}$, according to standard arguments regarding the asymptotic level density [13]. This is contained in the explicit factor of d_r in (4.9). Therefore, unless the couplings $G_{\ell m}^i$ both decrease exponentially rapidly as $\omega \rightarrow \infty$, the amplitude will exhibit exponential behavior.

Of course, such a degeneracy factor occurs in other one loop string amplitudes, and doesn’t produce exponential enhancement. One example is in high energy string scattering [14,15] where we see Regge behavior. Another is in the case of decay of high-level leading Regge-trajectory states [16] where kinematical angular momentum constraints forbid emission of all but certain restricted states. However, these constraints don’t apply to the present case of an initial state on a highly subleading trajectory. Furthermore, the type of process being considered is very different from high-energy string scattering. In

high-energy string scattering, the softness of the string interaction presumably interferes with efficient excitation of the exponential degeneracy of available states – the energy is not easily transferred to world-sheet oscillations. However, here one is starting out with a state that is high-frequency on the worldsheet. This, and the effective hardness of the D-brane interaction, may allow one to access the exponentially growing number of states in this process.

Indeed, via a rather lengthy analysis it is possible to find an approximation to $G_{\ell 0}^i$ that appears not to exhibit such exponential suppression, although this analysis is not conclusive as it is not inconceivable that cancellations with the other $G_{\ell m}^i$'s could produce such suppression.

Therefore, it seems at least plausible that instead of (5.1) we have

$$\text{Im } \mathcal{A} \propto g^2 e^{c\omega}. \quad (5.2)$$

This would mean that the scale at which the scattering becomes appreciable is the string scale, or at most logarithmically down from the string scale. If this is the case, scattering ripples from branes will not necessarily reveal the desired sub-stringy structure.

Confirmation of this (or the more interesting converse) requires the application of more clever techniques to the expression (3.17). Another approach in the literature [17] involves getting decay rates from asymptotic analysis at *large* λ , but so far (3.17) has not proven amenable to such treatment. Perhaps related or other techniques will ultimately allow a check of the conjecture (5.2).

ACKNOWLEDGMENTS:

I would like to thank J. Polchinski for interesting me in this problem and for suggestions, and S. Chaudhuri for discussions. Parts of this work was completed at the Aspen Center for Physics. This work was supported in part by NSF PYI grant nsf-phy9157463 and by grant DOE91ER40618.

Appendix A.

This appendix contains a brief review/summary of some useful one-loop technology.

A.1. Green functions on annuli

One needs explicit expressions for Green's functions on an annulus of circumference 1 and height λ . First consider the case of the torus. Since the function

$$\theta_1(z|\tau) = i \sum_{n=-\infty}^{\infty} (-1)^n q^{(n-1/2)^2} e^{i\pi(2n-1)z} \quad (\text{A.1})$$

vanishes at $z = 0$, a natural guess for the field of a point charge at w is

$$\ell n \theta_1(z - w|\tau). \quad (\text{A.2})$$

However, this is not single valued under $z \rightarrow z + 1$ and $z \rightarrow z + \tau$, under which

$$\begin{aligned} \theta_1(z + 1|\tau) &= -\theta_1(z|\tau), \\ \theta_1(z + \tau|\tau) &= -e^{-i\pi\tau - 2i\pi z} \theta_1(z|\tau). \end{aligned} \quad (\text{A.3})$$

This is related to the fact that it is not possible to place a single charge on a closed surface and obey Gauss's law. The solution is to introduce a background charge so that the net charge is zero. There are many ways to do this, but one is simply to put the background charge at a point, $z = w'$. One still has to add a homogeneous solution of Laplace's equation to satisfy all periodicity conditions, and the resulting (chiral) Green function is

$$G_T(z; w, w'|\tau) = \frac{1}{2} \ell n \left[\frac{\theta_1(z - w|\tau)}{\theta_1(z - w'|\tau)} \right] - \frac{\pi i (w - w') \text{Im} z}{\text{Im} \tau}. \quad (\text{A.4})$$

Combining this with a similar expression involving \bar{z}, \bar{w} , etc., gives the full scalar Green function with the normalization convention $\nabla^2 G = 2\pi\delta$. An important consistency check on amplitudes (momentum conservation) is that dependence on the arbitrary point w' drops out.

The Green functions for the annulus are easily constructed using the fact that the double of the annulus, with modulus λ , along its boundaries is the torus with modulus $\tau = 2i\lambda$. Thus, we simply place image charges to satisfy the appropriate boundary conditions. For Neumann boundary conditions, $(\partial_z - \partial_{\bar{z}})G_N(z, w, w'|\lambda) = 0$ at $\text{Im} z = 0$, λ is satisfied by

$$G_N(z; w, w'|\lambda) = \ell n \left| \frac{\theta_1(z - w|2i\lambda)\theta_1(z - \bar{w}|2i\lambda)}{\theta_1(z - w'|2i\lambda)\theta_1(z - \bar{w}'|2i\lambda)} \right|, \quad (\text{A.5})$$

which is easily seen to have the required periodicity under $z \rightarrow z + 1$. For Dirichlet boundary conditions $G_D(z, w|\lambda) = 0$ at $\text{Im} z = 0, \lambda$, here no background charge is needed, and we find

$$G_D(z, w|\lambda) = \ell n \left| \frac{\theta_1(z - w|2i\lambda)}{\theta_1(z - \bar{w}|2i\lambda)} \right| - \frac{2\pi i (w - \bar{w}) \text{Im} z}{\text{Im} \tau}. \quad (\text{A.6})$$

A.2. Functional determinants on annuli

The determinant of $\Delta = -\nabla^2$ on an annulus with either Neumann or Dirichlet boundary conditions is easily written in terms of evaluable infinite products. With Neumann boundary conditions, eigenfunctions are

$$\chi_{m,n} = e^{2\pi i n \sigma^1} \cos\left(\frac{m\pi}{\lambda} \sigma^2\right), \quad (\text{A.7})$$

and with Dirichlet boundary conditions,

$$\psi_{m,n} = e^{2\pi i n \sigma^1} \sin\left(\frac{m\pi}{\lambda} \sigma^2\right). \quad (\text{A.8})$$

These yield

$$\det'_N \Delta = \left(\prod_{n=1}^{\infty} 4\pi^2 n^2 \right)^2 \prod_{m=1}^{\infty} \frac{m^2 \pi^2}{\lambda^2} \prod_{n=1}^{\infty} \left(\frac{m^2 \pi^2}{\lambda^2} + 4\pi^2 n^2 \right)^2 \quad (\text{A.9})$$

and

$$\det_D \Delta = \prod_{m=1}^{\infty} \frac{m^2 \pi^2}{\lambda^2} \prod_{n=1}^{\infty} \left(\frac{m^2 \pi^2}{\lambda^2} + 4\pi^2 n^2 \right)^2, \quad (\text{A.10})$$

where prime indicates omission of the zero mode. These infinite products are readily evaluated using identities given in, for example, [18].

In particular,

$$\prod_{n=1}^{\infty} a = \frac{1}{\sqrt{a}}, \quad (\text{A.11})$$

for arbitrary constant a , and

$$\prod_{n=1}^{\infty} n^2 = 2\pi, \quad (\text{A.12})$$

so the two determinants are equal. Further application of such identities (see [18]) gives

$$\det'_N \Delta = \det_D \Delta = 2\lambda \left[e^{-\frac{\pi\lambda}{6}} \prod_{n=1}^{\infty} \left(1 - e^{-4\pi\lambda n} \right) \right]^2. \quad (\text{A.13})$$

References

- [1] J. Polchinski, S. Chaudhuri, and C.V. Johnson, “Notes on D-branes,” hep-th/9602052; J. Polchinski, “TASI lectures on D-branes,” hep-th/9611050, to appear in the proceedings of the 1996 Theoretical Advanced Study Institute.
- [2] Useful reviews are J.H. Schwarz, “Lectures on superstring and M theory dualities,” hep-th/9607201, to appear in the proceedings of the 1996 Theoretical Advanced Study Institute;
M.R. Douglas, “Superstring dualities, Dirichlet branes, and the small scale structure of space,” hep-th/9610041, to appear in the proceedings of the Les Houches School on Theoretical Physics, Session 64, Quantum Symmetries, Les Houches, France, 1995.
- [3] C. Vafa, talk at 1996 CERN Workshop on S-duality.
- [4] T. Banks, W. Fischler, S.H. Shenker, and L. Susskind, “M theory as a matrix model: a conjecture,” hep-th/9610043.
- [5] T. Banks and L. Susskind, “Brane – anti-brane forces,” hep-th/9511194.
- [6] S.H. Shenker, “Another length scale in string theory?” hep-th/9509132.
- [7] I.R. Klebanov and L. Thorlacius, “The size of p-branes,” hep-th/9510200, *Phys. Rev. Lett.* **B371** (1996) 51.
- [8] W. Fischler, S. Paban, and M. Rozali, “Collective coordinates for D-branes,” hep-th/9604014, *Phys. Lett.* **B381** (1996) 62 ;
D. Berenstein, R. Corrado, W. Fischler, S. Paban, and M. Rozali, “Virtual D-branes,” hep-th/9605168, *Phys. Lett.* **B384** (1996) 93.
- [9] C. Bachas, “D-brane dynamics,” hep-th/9511043, *Phys. Lett.* **B374** (1996) 37.
- [10] M.R. Douglas, “Gauge fields and D-branes,” hep-th/9604198.
- [11] D. Kabat and P. Pouliot, “A comment on zero-brane quantum mechanics,” hep-th/9603127, *Phys. Rev. Lett.* **77** (1996) 1004;
M.R. Douglas, D. Kabat, P. Pouliot, and S.H. Shenker, “D-branes and short distances in string theory,” hep-th/9608024.
- [12] E. Del Giudice, P. DiVecchia, and S. Fubini, “Factorization and operator formalism in the generalized Veneziano model,” *Nuov. Cim.* **5A** (1971) 90.
- [13] See e.g. M.B. Green, J.H. Schwarz and E. Witten, *Superstring Theory*, Chapter 2.
- [14] D.J. Gross and P.F. Mende, “The high-energy behavior of string scattering amplitudes,” *Phys. Lett.* **197B** (1987) 129; “String theory beyond the Planck scale,” *Nucl. Phys.* **B303** (1988) 407.
- [15] D. Amati, M. Ciafaloni, and G. Veneziano, “Superstring collisions at Planckian energies,” *Phys. Lett.* **197B** (1987) 81.
- [16] V. Balasubramanian and I.R. Klebanov, “Some aspects of massive world brane dynamics,” hep-th/9605174, *Mod. Phys. Lett.* **A11** (1996) 2271.

- [17] D. Mitchell, N. Turok, R. Wilkinson, and P. Jetzer, “The decay of highly excited open strings,” *Nucl. Phys.* **B315** (1989) 1 ;
D. Mitchell, B. Sundborg, and N. Turok, “Decays of massive open strings,” *Nucl. Phys.* **B335** (1990) 621.
- [18] S.B. Giddings, “Fundamental strings,” in the proceedings of the 1988 Theoretical Study Institute, A. Jevicki and C.-I Tan, eds. (World Scientific, 1989).